ETU J. Math., 1 (1) (2023) 1-7

Erzurum Technical University Journal of Mathematics

Journal Homepage: https://dergi.erzurum.edu.tr/etujm
e-ISSN:…

The n^{th} Root of a Complex Number

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Abstract

In this study, a general formula has been obtained that gives the area of a geometric figure that accepts n^{th} order roots of a complex number *z=r.cisθ* in complex numbers. This formula was generalized using the limit rules for $n \to \infty$ and it was seen that it reached π, which is the area of the unit circle.

Keywords: *Complex number,* n^{th} *root.*

2010 Mathematics Subject Classification: *30A99*

1. Introduction

It is a very important process to find the root of a complex number or matrix. Therefore there are several studies about it in the literature about root finding of a complex number. In [1], a new and fast computational method has been proposed and it is shown that the convergence rate of this new method is very high when comparison with the literature methods. The extraction method of the real or complex elements 2×2 matrices has been proposed in [2]. In [5], a new and very useful general methodology and architecture to compute n^{th} root of a complex number approximately using the coordinate rotation digital computer (CORDIC) has been proposed at the first time in literature. The computation of the n^{th} root of a complex number with the 2.16×10^{-6} error using a new methodology and architecture with low-latency and low complexity has been given in [6].
We used certain known formulas in the problems of finding n^{th} roots of complex numbers. We encountered the questions of geometric shapes that accept these roots as vertices and finding the area of these shapes. We could easily solve the questions for small values of n. However, we found that we had difficulties as n got larger and tried to find a general rule for each value of n. In general, we obtained the formula of the geometric figure that accepts the n^{th} -order roots of the number $z = r.cis\theta$ as vertices and applied it to various questions. Also, in this formula, we have reached the area formula of the circle formed for $n \to \infty$. While rules for finite values of n were found in previous research projects in this field, generalization was made for infinite values of n in this study. Most importantly, the number π, which has an important place in mathematics,

Article history:
Received: 15 November 2022
Accepted: 30 January 2023

has been reached.

2. Main Results

Theorem 2.1. If the area of the geometric figure that accepts the n^{th} -order roots of the number *z=r.cisθ* as vertices is *A*

$$A = n.\frac{1}{2}.\sqrt[n]{r^2}.sin\frac{2\pi}{n} \quad (n>2) \tag{2.1}$$

Proof.
The n^{th} roots of the number *z=r.cisθ*

$W_K = \sqrt[n]{r}.\text{cis}\left(\frac{\theta+2k\pi}{n}\right)$ *k=1,2,3,4,…,n-1* .

Where

$$k = 0 \;=>\; W_0 = \sqrt[n]{r}.cis\left(\frac{\theta}{n}\right)$$

$$k = 1 \;=>\; W_1 = \sqrt[n]{r}.cis\left(\frac{\theta}{n}+\frac{2\pi}{n}\right)$$

$$k = 2 \;=>\; W_3 = \sqrt[n]{r}.cis\left(\frac{\theta}{n}+\frac{4\pi}{n}\right)$$

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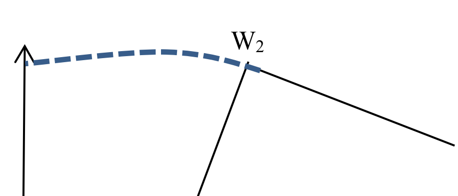
$$k = n-1 \;=>\; W_{n-1} = \sqrt[n]{r}.cis\left(\frac{\theta}{n}+\frac{2(n-1)\pi}{n}\right)$$

$Arg(W_k) - Arg(W_{k-1}) = \frac{2\pi}{n}$



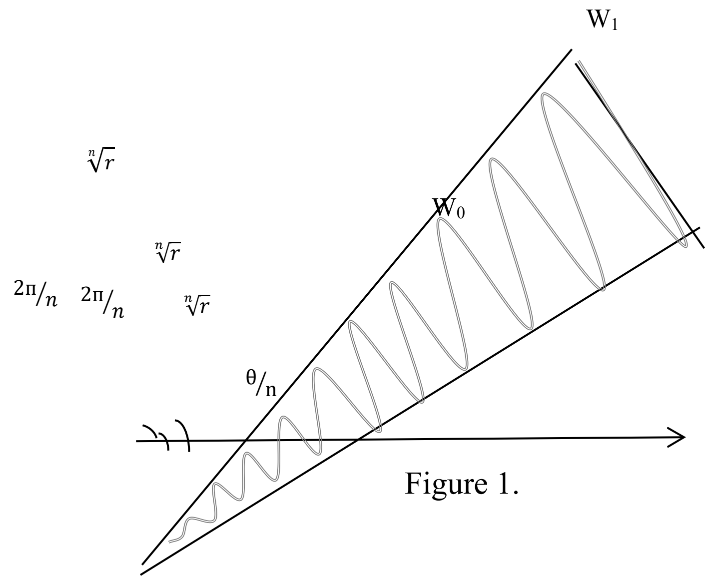


Figure 1.

$A\ (w1\hat{0}w0) = \frac{1}{2}.\sqrt[n]{r}.\sqrt[n]{r}\ sin\frac{\pi}{2}$ $(sinus\ theorem)$ [3,4]

$$A\ (w1\hat{0}w0) = \frac{1}{2}.\sqrt[n]{r^2}.sin\frac{\pi}{2}$$

The area of the geometric figure that accepts all roots as vertices is *n* times A (w1$\hat{0}$w0).

Then,

All area= $n.\frac{1}{2}.\sqrt[n]{r^2}$.sin$\frac{\pi}{2}$

Let's generalize our theorem for $n \rightarrow \infty$:

Let's show the *nth*-order roots of the number $z = r.cis\theta$ with the area function $A(n)$ of the geometric figure that accepts the vertex.

$\sqrt[n]{r}$=r_1 then,

$$A(n) = \frac{n}{2}.r_1^2.sin\frac{2\pi}{2}$$

If the limit is taken for $n \rightarrow \infty$

$A(n) = \lim_{n\rightarrow\infty} (\frac{n}{2}.r_1^2.sin\frac{2\pi}{2}) = \infty.0$ uncertainty.

$A(n) = \lim_{n\to\infty} \frac{r_1^2 . sin\frac{2\pi}{2}}{\frac{n}{2}}$

$\frac{1}{n} = t$ when $n \to \infty$ then $t \to \infty$.

$A(n) = \lim_{t\to\infty} \frac{r_1^2 . sin 2\pi t}{2t}$ $\quad (\lim_{x\to\infty} \frac{\sin ax}{bx} = \frac{a}{b}.)$

$A(n) = r_1^2 \frac{2\pi}{2}$ then

$A(n) = \pi . r^2.$

EXAMPLES

Example 1.

Find the area of the geometric figure that considers the 6^{nth} degree roots of the number $z = 64cis\frac{\Pi}{3}$ as vertices?

Solution:

$$A = \frac{n}{2} . \sqrt[n]{r^2} . sin\frac{2\pi}{n}$$

$$A = \frac{6}{2} . \sqrt[4]{81^2} . sin\frac{2\pi}{4}$$

$$A = 3.4\, sin\frac{\pi}{3}$$

$$A = 12. \frac{\sqrt{3}}{2}$$

$$A = 6. \sqrt{3}\, br2$$

Example 2.

Find the area of the geometric figure that accepts the 4^{th} degree roots of the number $z = 81. cis400$ as vertices?

Solution:

$$A = \frac{n}{2}.\sqrt[n]{r^2}.sin\frac{2\pi}{n}$$

$$A = \frac{4}{2}.\sqrt[4]{81^2}.sin\frac{2\pi}{n}$$

$$A = 2.9.sin\frac{\pi}{2}$$

$$A = 18.1$$

$$A = 18\ br^2$$

Example 3.

Find the area of the geometric figure that accepts the 9^{th} roots of the number $z = cis9°$ as vertices?
Solution:

$$A = \frac{n}{2}.\sqrt[n]{r^2}.sin\frac{2\pi}{n}$$

$$A = \frac{9}{2}\sqrt[9]{1^2}.sin\frac{2\pi}{9}$$

$$A = \frac{9}{2}.sin\frac{2\pi}{9}\ \text{br}^2.$$

Problem 4.

Find the area of the geometric figure whose 4^{th} order roots of $z = 128.cis\ \pi/3$ are considered vertices?

Solution:

$$A = \frac{n}{2}.\sqrt[n]{r^2}.sin\frac{2\pi}{n}$$

$$A = \frac{4}{2}.\sqrt[4]{128^2}.sin\frac{2\pi}{n}$$

$$A = 2.8\sqrt{2}.1$$

$A = 16\sqrt{2}$ br^2.

Result 1.

We show that the area of the geometric figure that accepts the n^{th} roots of the number $z = r\ cis\theta$ as vetices is:

$$A = n.\frac{1}{2}.\sqrt[n]{r^2}.sin\frac{2\pi}{n}$$

Result 2.

We find that the geometric shape formed for $n \to \infty$ is a circle and its area:

$A(n) = \pi.r_1^2$

where

$\sqrt[n]{r}=r$.

3. Conclusion

We have suggest a formula to obtain the are of a geometric figure which accepts the n^{th} root of a complex number.

Acknowledgement

The author thanks the editors and reviewers for their valuable contributions and suggestions to improve the paper. In the revised version, all the reviewers' comments were taken into consideration, resulting in a substantial improvement with respect to the original submission.

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